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TEMPERATURE AND CONCENTRATION PROFILES IN FULLY TURBULENT BOUNDARY LAYERS

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NOMENCLATURE

a, thermal diffusivity;
c_p, heat capacity at constant volume;
Pr, $= v/a$, Prandtl number;
q_w, wall heat flux;
R, radius of tube;
t, temperature;
t_*, $= q_w/\rho c_p u_*$, characteristic temperature;
u_*, friction velocity;
y, coordinate normal to the wall;
y_+, $= yu_*/v$, dimensionless coordinate;
δ, boundary layer thickness;
θ_+, $= (t_w - t)/t_*$, dimensionless temperature;
ε, eddy diffusivity;
v, kinematic viscosity;
ρ, density.

Subscripts

w, conditions at the wall;
o, external conditions.

CONSIDER a temperature or concentration field in a fully turbulent boundary layer of thickness δ developed along a smooth plate or a smooth wall of a tube of radius R. In the following, for clarity I shall speak of temperature bearing in mind that all of the results are also valid for the concentration profile with respective changes in notation. Note also that in the subsequent analysis the assumption will be made that heat is a passive admixture, i.e. it does not influence the flow dynamics.

Two different approaches to calculation of the temperature profile $t(y)$ are used in the available literature. The first approach, developed by Reichardt [1], Deissler [2] and some other authors, is based on direct numerical integration of the differential equation

$$d\theta_+/dy_+ = (Pr^{-1} + \varepsilon/v), \quad \theta_+(0) = 0; \quad \theta_+(\delta), \quad \theta_+(R) = \theta_{+0} \quad (1)$$

$$\theta_+ = \frac{t_w - t}{t_*}, \quad \theta_{+0} = \frac{t_w - t_o}{t_*}, \quad t_* = \frac{q_w}{\rho c_p u_*}, \quad y_+ = \frac{yu_*}{v}.$$

Here t_* is the friction temperature determined by the wall heat flux q_w, fluid density ρ, heat capacity c_p and the friction velocity u_*; t_o is the temperature at the tube center line or outer edge of the boundary layer; $Pr = v/a$ the molecular Prandtl number and ε the eddy diffusivity of heat. Equation (1) determines the temperature profile near the wall of temperature t_w in the case when longitudinal variations of the temperature field are negligible as compared with its variations along the y-axis normal to the wall.

Different authors suggested different equations for $\varepsilon = \varepsilon(y, Pr)$ but all these are very cumbersome and this rules out the possibility of obtaining simple analytical relationships for $\theta_+(y)$.

The second approach is based on the choice of an interpolation formula which agrees, with tolerable accuracy, with the observed behavior of the temperature profile in the molecular transport sublayer, logarithmic sublayer and outer part of the boundary layer. This idea is at the basis of the equations suggested by Petukhov, Polyakov and Kuleshov [3] and Snijders *et al.* [4] for gases ($Pr \sim 0.7$) and by Kalbaliev [5] for the range of Pr numbers from 0.02 to 80. However, none of the relations suggested satisfies the whole set of conditions required to obtain a good approximation of a temperature profile.

Let us enumerate these conditions which follow from the common physical ideas on the mechanism of turbulent heat transfer in the boundary layer.

(1) In the immediate vicinity of the wall the coefficient of turbulent transfer decays rapidly and tends to zero as $y \to 0$. Hence, a molecular transport sublayer adjacent to the wall exists in which the molecular heat transfer dominates. Temperature distribution in this zone can be found from equation (1) where $\varepsilon = 0$ so that

$$\theta_+ = Pr y_+. \quad (2)$$

(2) The dimensionless thickness y_1 of this molecular transport sublayer can be evaluated with the aid of the condition $\varepsilon(y_1) \cong a$. When $Pr \gg 1$, the molecular heat conduction sublayer is deeply immersed in the viscous sublayer and the value of ε is represented with a good accuracy by an approximate power formula of the form $\varepsilon/v = by_+^m$. The data of statistical treatment of the most reliable electrochemical experiments [6] suggest the assumption that $b = 6 \times 10^{-4}$ and $m = 3$ for $500 \lesssim Pr \lesssim 40 \times 10^3$. Therefore

$$y_1 \cong 12/Pr^{1/3} \quad \text{at } Pr \gg 1. \quad (3a)$$

For gases ($Pr \sim 1$) the velocity and temperature fields are approximately similar. Hence

$$y_1 \cong 30 \quad \text{at } Pr \sim 1. \quad (3b)$$

When $Pr \ll 1$, the sublayer of molecular heat conduction spreads from the wall to the region where the logarithmic velocity distribution is valid and $\varepsilon/v = Pr_t/(ky_+)$, where $k \cong 0.4$ is the von Karman constant. According to [7], the turbulent Prandtl number in this zone is given by the relation $Pr_t \cong 0.85$. Thus

$$y_1 \cong 2/Pr \quad \text{at } Pr \ll 1. \quad (3c)$$

(3) Suppose that the following generally accepted assumption is valid: both the coefficients of eddy viscosity and eddy thermal diffusion are determined by velocity and temperature fluctuations only and do not depend on a, i.e. Pr_t is independent of Pr (though it can vary with y in the wall region). Then the coefficients of Taylor expansion of ε of the form $\varepsilon/v = a_3 y_+^3 + a_4 y_+^4 + \ldots$ must be universal constants. In this case equation (1) [8] immediately yields

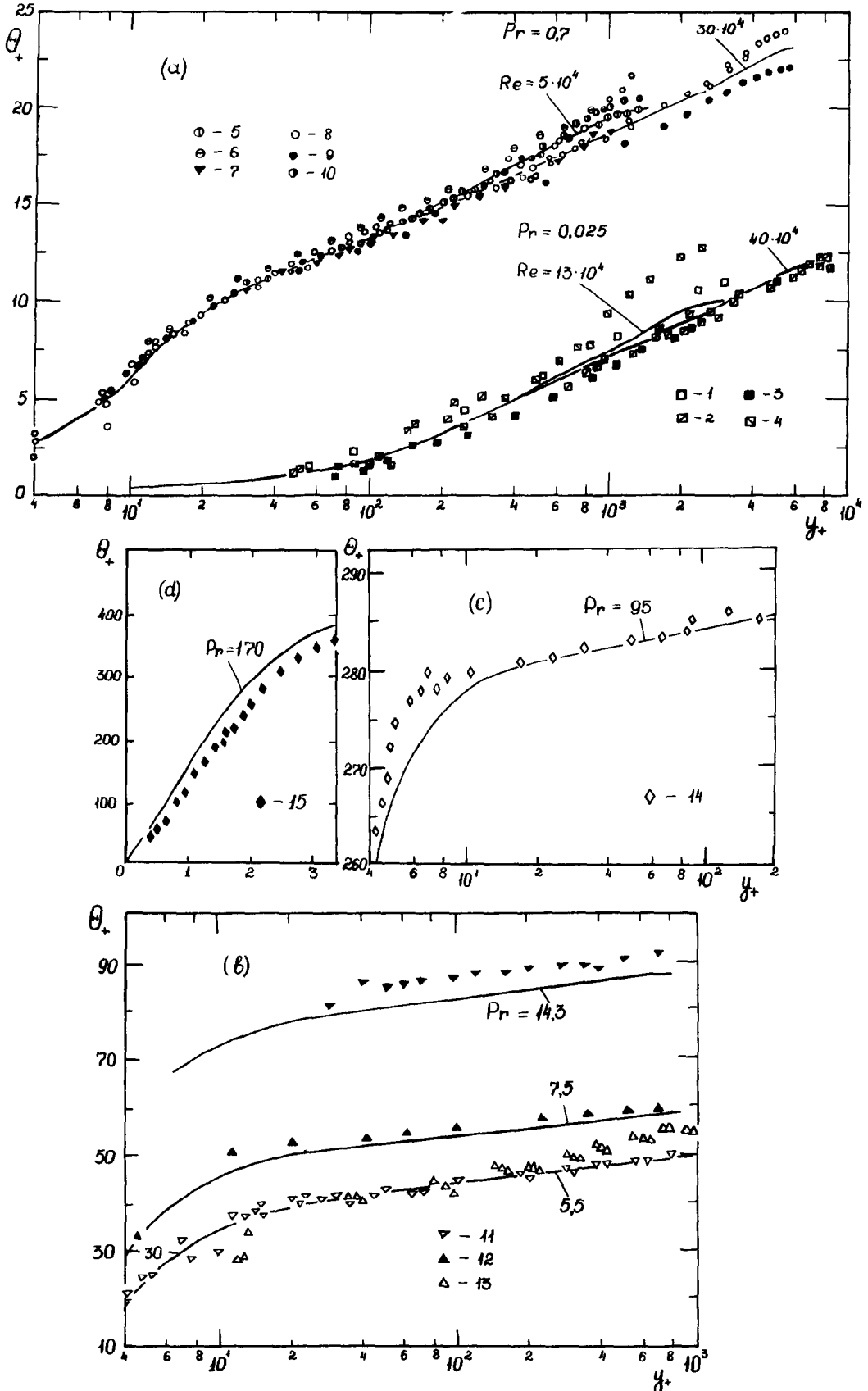$$\theta_+ = Pr y_+ - Pr^2(c_4 y_+^4 + c_5 y_+^5 + \ldots) \quad (4)$$

FIG. 1. Temperature profiles in a turbulent flow in pipes and channels of liquid metal and air (a), water and ethylene glycol (b), technical oil (c) and (d) based on the experimental data of : 1, [9, 10] ; 2, [11] ; 3, [12] ; 4, [13] ; 5, [14] ; 6, [15] ; 7, [16] ; 8, [17] ; 9, [18] ; 10, [19, 20] ; 11, [21] ; 12, [22] ; 13, [23] ; 14, [24] ; 15, [25]. Solid lines correspond to equation (9)
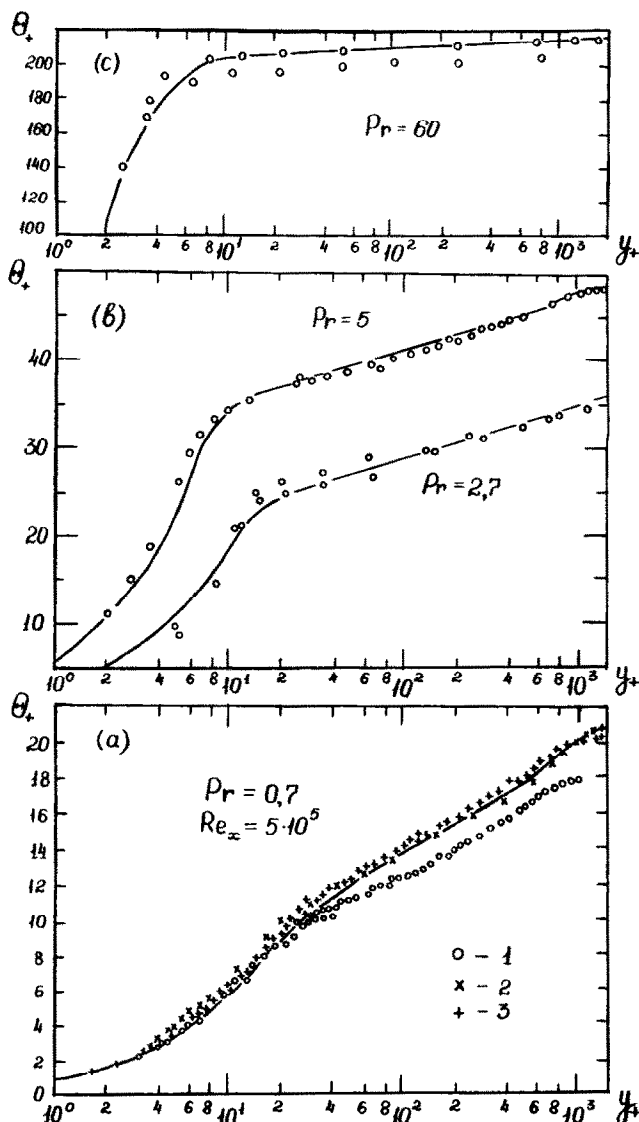
FIG. 2. Temperature profiles in a turbulent flow of air (a), water (b) and transformer oil (c) on the plate based on the experimental data of: 1, [26]; 2, [27]; 3, [28]. Solid lines correspond to equation (10).

where, according to the above estimates, $a_3 \cong 6 \times 10^{-4}$ and, hence, $c_4 = a_3/4 \cong 1.5 \times 10^{-4}$. Thus, appreciable (within 10%) deviations from the linear temperature profile (2) at $Pr > 1$ begin at $y_+ \sim 9/Pr^{1/3}$. This value agrees well with both the available experimental data and the estimate (3a).

(4) In the region $y_1 \ll y_+ \ll \delta_+$ (or R_+) the temperature distribution obeys the logarithmic law

$$\theta_+ = \alpha \ln y_+ + \beta(Pr). \tag{5}$$

As is shown in [7], this result can be deduced from general similarity arguments using no assumptions related to ε. Moreover $\alpha \cong 2.12$ and, hence, $Pr_t = \text{const} \cong 0.85$. Some additional arguments were used in [7] to establish an approximate form of the function $\beta(Pr)$ which determines the temperature difference between the wall and the lower edge of the logarithmic layer. Revision of the values of empiric coefficients in the equation for $\beta(Pr)$ on the basis of the most reliable recent experimental data leads to the following relationship

$$\beta(Pr) = (3.85 Pr^{1/3} - 1.3)^2 + 2.12 \ln Pr \tag{6}$$

which holds for the range of Pr numbers from 6×10^{-3} to 40×10^3.

(5) Dimensional arguments lead to the conclusion that in a fully turbulent flow, at $0.1 \lesssim y/\delta, y/R \lesssim 1$, the molecular fluid properties do not influence the dimensionless temperature distribution, so that

$$\theta_+ = f_1(y/\delta) \quad \text{or} \quad \theta_+ = f_2(y/R). \tag{7}$$

It is also evident that

$$\theta'_+(\delta) = \theta'_+(R) = 0. \tag{8}$$

All of the above conditions are met by the following simple equations

$$\theta_+ = Pr\, y_+ \exp(-\Gamma)$$
$$+ \left\{ 2.12 \ln \left[(1 + y_+) \frac{1.5(2 - y/R)}{1 + 2(1 - y/R)^2} \right] + \beta(Pr) \right\} \exp(-1/\Gamma) \tag{9}$$

for tube or channel flow and

$$\theta_+ = Pr\, y_+ \exp(-\Gamma)$$
$$+ \left\{ 2.12 \ln \left[(1 + y_+) \frac{2.5(2 - y/\delta)}{1 + 4(1 - y/\delta)^2} \right] + \beta(Pr) \right\} \exp(-1/\Gamma) \tag{10}$$

for boundary layer flow, where

$$\Gamma = \frac{10^{-2}(Pr\, y_+)^4}{1 + 5Pr^3 y_+}.$$

These equations are based on the interpolation arguments and permit evaluation of the temperature field in a tube or a plane channel (in the latter case R is the half-width of the channel) and in a boundary layer above a smooth plate at any value of Pr. Equations (9) and (10) are compared in Figs. 1 and 2 with the available experimental data on heat transfer in turbulent flows of mercury [9-12], NaK [13], air [14-20], [26-28], water [21-23, 26], ethylene glycol [16] and technical oil [24-26]. The figures show that the agreement between the predicted temperature distribution and all the experimental data is quite satisfactory.

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